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## NEUTRALINO-NUCLEON INTERACTION IN THE SPLIT SUSY SCENARIO OF THE DARK MATTER

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The Split SUSY scenario with light Higgsino states is treated as an application to the Dark Matter problem. We have considered the structure of the neutralino-nucleon interaction and calculated cross-section of the neutralino-nucleon scattering. The decay properties of the lightest chargino and next lightest neutralino are analyzed in details.

*Keywords:* neutralino-nucleon interaction; Split SUSY; Higgsino; Dark Matter

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### 1. Introduction

One of the most important phenomenological applications of the Supersymmetry (SUSY) is the treatment of neutralino as a candidate for the (Cold) Dark Matter (DM). In Refs. [1, 2, 3, 4, 5] new hierarchies of SUSY scales (so called Split Supersymmetry scenarios) were motivated, on the one hand, by the Anthropic Principle and, on the other hand, multi-vacua string landscape arguments. Scenarios of this type are alternative to the MSSM in some aspects and do not reject the fine-tuning mechanism in the spectrum of scales [6, 7, 8, 9, 10, 11, 12].

The Renormalization Group (RG) analysis of the Split SUSY models was performed in Refs. [3, 5, 6, 11, 12]. In Ref. [12] the one-loop RG behavior of SUSY  $SU(5)$  was considered in detail accounting for degrees of freedom in the vicinity of  $M_{GUT}$  scale. It was noticed that these heavy states should be considered as threshold corrections; they are important for the final arrangement of scales providing sufficiently high unification point. From the RG consideration at the one-loop level two classes of scenarios emerge having an opposite arrangement of  $\mu$  and  $M_{1/2}$  scales ( $M_{1/2}$  is an order of  $M_1$  or  $M_2$ ). At the same time, the analysis does not fix the characteristic scale of superscalars,  $M_0$ , due to a specific form of the one-loop equations  $\mu, M_{1/2} = f_a(M_{GUT})$  following from the RG analysis. More precisely, these equations contain the squark and slepton scales in ratios  $M_{\tilde{q}}/M_{\tilde{l}}$  only.

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For the first class of scenarios the hierarchy  $|\mu| \gg M_{1/2}$  takes place. The second class is defined by the opposite hierarchy  $|\mu| \ll M_{1/2}$ . In particular, the RG analysis [12] results in the hierarchy

$$M_0 \sim M_{1,2} \gg |\mu| > M_{EW}, \quad (1)$$

whose spectrum contains two lightest neutralinos degenerated in mass (almost pure Higgsino case) and one of charginos as the nearest to the electroweak scale  $M_{EW}$ . In this scenario both  $M_0$  and  $M_{1/2}$  can be shifted to scales  $\sim (10^6 - 10^{10})$  GeV. As it was shown in Ref. [10, 12], in one-loop approximation the RG does not fix these scales strictly due to uncertainties in dimensionless parameters, which are defined by the heavy scales ratios. Usually assumed value for this scale is  $\sim 10^8 - 10^9$  GeV and it is within our RG motivated interval.

In this paper, we study some particular features of the scenario (1) and their consequences in experiments. We consider the basic characteristics of the neutralino manifestations – the cross-section of the neutralino-nucleon scattering and decay properties of the neutralino and chargino. We will explicitly show that the direct observation of relic neutralino (Higgsino) in the  $\chi - N$  scattering is impossible due to the fact that the typical energy of relic  $\chi$  is far below the corresponding threshold (see Section 2). So, we concentrate our attention on the high-energy neutralino-nucleon scattering and decays of its products.

The program of collider experiments is mainly based on study of the MSSM, which has supersymmetric degrees of freedom near 1 TeV. At this scale the Split Supersymmetry displays the isolated lightest scales only. In particular, in the scenario under consideration there are two lightest neutralino (LSP,  $\chi_1^0$  and NLSP,  $\chi_2^0$ ) and one light chargino  $\tilde{H}$  with the masses  $\sim 1$  TeV.

The signature of the neutralino and chargino production and decays at the LHC was considered in many papers. It was shown that this signature crucially depends on values of the mass splitting [11], [13]-[23]. In these papers main attention was paid to calculation of the cross-section of neutralino and chargino production at hadron colliders. The radiative decays at the loop level were considered in Ref. [14]. Decay rates in semileptonic and hadronic (production of quarks and jets) decay channels of neutralino and chargino were discussed there in some detail. It was pointed out that, on the one hand, observation of these hardly detectable effects would mean the possibility to gain important information about the scales of higher SUSY states (in particular,  $\tilde{t}_{1,2}$  and their mixing) just from the one-loop mass splitting calculations [24]-[30]. On the other hand, if only low-lying neutralino and chargino are detected in experiments near TeV scale, the conventional MSSM spectrum should be necessarily split in some manner to produce higher scales for other superstates. The spectrum of the lowest states,  $\chi_{1,2}$  (LSP and NLSP) and  $\tilde{H}$  (chargino), is nearly degenerated. For the scenario (1) this fact is well known (see, for example, Refs. [30]-[34]). Two other neutralino states,  $\chi_{3,4}$ , and heavy chargino  $\tilde{W}$  are placed far from the lightest ones at the scale  $\sim M_{SUSY}$ .

According to the well-known method and results of the relic abundance analysis

(Refs. [35]–[41] and references therein) neutralinos before their freeze-out live in the thermodynamical equilibrium with other components of the cosmological plasma. In order to compare the calculated value of relic abundance  $\Omega h^2$  with the corresponding experimental corridor given by the relic data [42], we have used the known values of SUSY parameters and extracted the following LSP (Higgsino) mass [10, 12]:  $M_\chi = 1.0 - 1.4$  TeV for  $x_f = 25$  and  $M_\chi = 1.4 - 1.6$  TeV for  $x_f = 20$ . These values do not break the gauge coupling convergence and are in good agreement with the results of Refs. [3, 4, 5, 11, 13]. Thus, in the model where these two lightest neutralinos and one chargino are closest to the EW scale, they have masses  $O(1$  TeV). Further, we will use  $M_\chi = 1.4$  TeV as an average value for all numerical estimations.

In this paper, we consider the possibility of registration of neutralino-nucleon scattering, when neutralinos are low-energy (relic) and high-energy (non-relic) ones. We show that in the first case the process is closed in the framework of the scenario under consideration. We calculate cross-section of the high-energy neutralino-nucleon scattering with the production of the lightest chargino and NLSP. We also consider in detail the decay rates of these products for the kinematically allowed channels.

The structure of the paper is as follows. In Section 2, the cross-section of neutralino-nucleon scattering is considered for low- and high-energy neutralino. The decay properties of the lightest chargino  $\tilde{H}$  and NLSP  $\chi_2$  are analyzed in Section 3 and 4. In these sections we describe the results of calculations, which are necessary for discussion of possible experimental manifestations of the considering SUSY scenario. Finally, some conclusions are given in Section 5. Appendices A and B contain an important details needed for calculations.

## 2. The neutralino-nucleon scattering

Supposing that the lightest neutralino  $\chi_1$  is the main DM constituent, experimental manifestations of the Split Higgsino scenario crucially depend on the neutralino mass splitting parameters  $\delta m = M_{\chi_2} - M_{\chi_1}$  and  $\delta m^- = M_{\tilde{H}} - M_{\chi_1}$ . These mass splittings are determined by the sum of their tree values and radiative corrections. Tree level mass splitting is approximately defined as [14, 31]:

$$\delta m \approx M_Z^2/M, \quad \delta m^- \approx 2\delta m^-, \quad (2)$$

where  $M \sim M_{1,2}$  and we suppose  $\tan\beta \gg 1$ . So, in the interval  $M \sim (10^6 - 10^{10})$  GeV the splitting is rather small:  $\delta m \sim (0.001 - 10)$  MeV. In the scenario under consideration, one-loop diagrams with  $\gamma$ ,  $Z$  and  $W$  bosons in the intermediate state contribute mainly to the value  $\delta m^-$ . Calculations of this contribution were performed in Ref. [10, 14] in various ways, and the same result was obtained,  $\delta m^- \approx 350$  MeV. To evaluate the contribution of the gauge bosons to  $\delta m$ , we should take into account the diagonal and non-diagonal self-energy contributions to the mass matrix [34]. However, this problem was not considered in detail for our case. It is

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usually assumed that loop corrections from the heavy  $\tilde{t}_a$  and  $\tilde{b}_a$ ,  $a = 1, 2$  states (they are all at the high  $M_0$  scale) to the mass splittings are negligible in such “low- $\mu$ ” scenarios. Note, however, that condition of the splitting smallness depends on the structure of these high energy states. If there is a significant gap in the superscalars mass spectrum, loop corrections both to  $\delta m$  and  $\delta m^-$  can be comparable with their tree values or even exceed them [15, 16, 24, 25, 26, 27, 28, 30, 33, 34]. As it is known [30], the hierarchy of  $\tilde{t}_1$  and  $\tilde{t}_2$  states and their mixing angle  $\theta_t$  drive the value of the mass difference when squarks dominate in loops (this simple approximation takes place when  $m_{\tilde{t}_1}^2 \gg m_{\tilde{t}_2}^2$ ):

$$\delta m \approx 2G_t^2 m_t \sin(2\theta_t) \cdot \ln\left(\frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2}\right), \quad (3)$$

where

$$G_t = \sqrt{\frac{3G_F}{8\sqrt{2}\pi^2}} \frac{m_t}{\sin\beta}. \quad (4)$$

For  $\sin\beta \approx 1$  and  $\sin 2\theta_t \approx 1$ , from Eq. (3) it follows  $\delta m \approx 5 \log(m_{\tilde{t}_1}^2/m_{\tilde{t}_2}^2)$  GeV. So, despite of heaviness of scalars in this scenario, their contribution to the mass splittings can be large if, for example, the ratio  $m_{\tilde{t}_1}/m_{\tilde{t}_2} \gtrsim 3$ . Because of lack of information about the hierarchy of the squark masses, mixing angle  $\theta_t$  and some details of loop contributions to the mass splittings within the Split SUSY scenarios, we assume  $\delta m, \delta m^- \lesssim 1 - 2$  GeV. Then, we will consider the decay properties of  $\tilde{H}$  and  $\chi_2$  in Sections 3 and 4 for these values of mass splittings. At any rate, this consideration (together with the analysis of possible experimental data on NLSP and chargino creation and decays) can provide some information on higher scale states in the split mass spectrum.

Here we discuss some theoretical possibilities for the direct detection experiments that can be given by the neutralino-nucleon interactions. This interaction structure strongly depends on the structure of the neutralino-boson interaction. Just this point demands very accurate mathematical analysis, which was performed in Ref. [31] (see also Appendix A). In our calculations we use  $L_{int}$  in the form [31]:

$$\begin{aligned} L_{int} = & g_2 W_\mu^+ \left( -\frac{i}{2} \bar{\chi}_2 \gamma^\mu \tilde{H} - \frac{1}{2} \bar{\chi}_1 \gamma^\mu \tilde{H} \right) + g_2 W_\mu^- \left( +\frac{i}{2} \tilde{\bar{H}} \gamma^\mu \chi_2 - \frac{1}{2} \tilde{\bar{H}} \gamma^\mu \chi_1 \right) \\ & + \frac{ig_2}{2 \cos\theta_W} Z_\mu \bar{\chi}_2 \gamma^\mu \chi_1, \end{aligned} \quad (5)$$

where the only light states are taken into account. Note, the important feature of the  $Z\chi\chi$  vertex which follows from the (5): the dominant contribution to this vertex is given by the non-diagonal vector-like term  $Z_\mu \bar{\chi}_2 \gamma^\mu \chi_1$ , while the axial vector terms  $Z_\mu \bar{\chi}_i \gamma^\mu \gamma_5 \chi_k$  are suppressed by the small mixing with heavy neutralino states. This fact directly follows from the connection between the neutralino mass sign, parity and structure of the neutralino-boson interaction [31]. Such interaction structure leads to the dominant contribution to the spin-independent (SI) part of

the neutralino-nucleon scattering in non-relativistic region (see later, Eq. (7) and Appendices A and B).

Processes of the lightest neutralino-nucleon scattering with  $\chi_2$  or  $\tilde{H}$  in the final state are presented in Fig. 1.

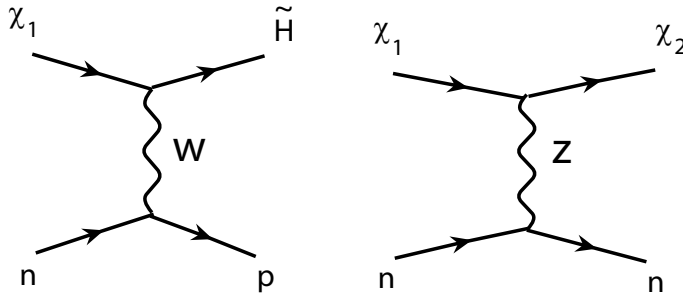


Fig. 1. Feynman diagrams for neutralino-nucleon scattering.

We consider two different cases of the process – the scattering of relic non-relativistic neutralino and the scattering of high-energy neutralino which can be produced by cosmic rays or in decays of an exotic  $M_X$  particles (for a review of such an option see, for example, [43, 44, 45]).

Neutralino-boson interaction in the considering scenario is described by the corresponding term in the Lagrangian (5). Effective low-energy interaction of nucleons and bosons is described by the vertex  $(ig/4 \cos \theta_W) \gamma_\mu (c_V - c_A \gamma_5)$ . Amplitude of the scattering is given by standard calculation rules:

$$M = \frac{ig^2}{8 \cos^2 \theta_W M_Z^2} \bar{\chi}_2 \gamma^\mu \chi_1 \cdot \bar{N} \gamma_\mu (c_V - c_A \gamma_5) N, \quad (6)$$

where  $N = n, p$ ,  $c_V = 1$  for neutron,  $c_V = 1 - 4 \sin^2 \theta_W$  for proton, and  $c_A \approx 1.25$  for both nucleons [38, 39]. We show that the relic neutralino can not overcome the threshold and there are no detectable signals at all. In contrast, the high energy neutralino overcome the threshold leading to a specific signature of the final states, which is considered below in detail.

Firstly, let us consider the relic low-energy neutralino typical for the (Cold) Dark Matter in the Galactic halo. In the case of pure Higgsino states in the framework of our scenario, the lowest order contributions to  $\chi - N$  interaction correspond to the spin-independent (SI) inelastic process [31]. It is a consequence of the Majorana formalism, where all neutralino have the same sign of masses, i.e. the same parity (see Ref. [31]). At the tree level, the cross-section for the relic neutralino-nucleon  $t$ -channel reaction in the non-relativistic limit mainly depends on the vector-like part of boson-nucleon interaction, which leads to the SI contribution. By the straightforward calculations one can get from Eq. (6) the cross-section in the following

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threshold form:

$$\sigma_{\chi N} = \frac{G_F^2}{4\sqrt{2}\pi} \frac{c_V^2}{v_r} M_N^{3/2} (W_\chi \frac{M_N}{M_\chi} - \delta m)^{1/2}, \quad (7)$$

where  $v_r$  is the dimensionless relative velocity in units of the speed of light  $c$ ,  $M_N$  is the nucleon mass, and the threshold term  $W_\chi(M_N/M_\chi) - \delta m$  directly follows from the kinematics of the process (see Appendix B for details). In the non-relativistic case  $W_\chi$  is an average kinetic energy of neutralino in the Sun neighborhood,  $W_\chi = M_\chi v_r^2/2$ . For  $M_{\chi_1} \sim 1$  TeV this energy is  $W_\chi \sim 1$  MeV. So, for  $\delta m > 1$  KeV the process is forbidden and the relic neutralino cannot be detected in the direct terrestrial experiments. The cross section for the neutralino-nucleon scattering with the chargino production (so called recharge process) is similar to Eq. (7), and for the splitting  $\delta m^- \approx 0.5 \cdot \delta m > 1$  KeV this channel is also closed. Note that the phenomenology of the relic neutralino-nucleon scattering crucially depends on the structure of the  $Z\chi\chi$ -interactions. The absence of the diagonal vertex (see Appendix A)  $Z\chi_1\gamma_5\gamma_\mu\chi_1$  (spin-dependent interaction) closes the direct channel of neutralino-matter interaction. This is a principal consequence of the analysis which have been made in Ref. [31].

Now we consider the scattering of the non-relic neutralino off nucleon above its threshold which is completely determined by the mass splittings  $\delta m$  and  $\delta m^-$ . As it was noted earlier, these values are  $\sim 10^2$  MeV. The energy of non-relic neutralinos can exceed the energy of the relic ones ( $\approx 1$  MeV) by many orders of magnitude. Such high-energy neutralinos can be produced all the time due to the non-elastic scattering of high-energy cosmic rays (or in various decay channels of super-heavy  $M_X$  particles). So, these high-energy neutralinos do not directly connected with the Cold Dark Matter.

The structure of the neutralino-chargino-boson vertices (5) allows s-channel transitions  $\tilde{f}\tilde{f} \rightarrow Z \rightarrow \chi_1\chi_2; \tilde{H}\tilde{H}$  and  $f_u f_d \rightarrow W \rightarrow \chi_{1,2}\tilde{H}$ . These processes can take place in collisions of high-energy particles from cosmic rays (or at the LHC), producing high-energy  $\chi_{1,2}$  and/or  $\tilde{H}$ . Further,  $\chi_2, \tilde{H}$  intensively decay producing the energetic LSP,  $\chi_1$ . As a result, some quantity of the lightest energetic LSPs, which are not relic ones, can be accumulated and exists now.

Let us consider the interactions of these neutralinos with matter, in particular, with nucleons. Obviously, the neutrino-nucleon scattering provides the main contribution to the background. Due to the fast decreasing of the cosmic rays density with the increase of energy, we restrict ourselves to the processes with the boson transverse momenta  $Q^2 \lesssim 2$  GeV. This restriction is consistent with the value of mass splitting and makes it possible to evaluate maximal cross-section and to apply the simplest vector and axial vector form-factors for the description of  $npW$  and  $NNZ$  vertices ( $N = n, p$ ). Amplitude of the recharge process  $\chi_1 n \rightarrow \tilde{H} p$  is

$$M = \frac{g^2}{4\sqrt{2}M_W^2} \bar{\chi}_1 \gamma^\mu \tilde{H} \cdot \bar{n} \gamma_\mu [c_V(Q^2) - c_A(Q^2)\gamma_5] p, \quad (8)$$

where  $c_V(0) = 1$  and  $c_A(0) \approx 1.25$ . The  $Q^2$ -dependence for the vector and axial-vector parameters can be chosen in the simplest form  $c_V(Q^2) = c_V(0)(1+Q^2/m_V^2)^{-2}$  and  $c_A(Q^2) = c_A(0)(1+Q^2/m_A^2)^{-2}$ , where  $m_V = 0.84$  GeV and  $m_A = 0.90$  GeV in analogy with the neutrino-nucleon scattering. In order to get a rough estimation of the cross-section, it is sufficient to use these two form-factors only. In this approximation, we get the cross-section of  $\chi_1 n \rightarrow \tilde{H} p$  scattering in the form:

$$\sigma(s) \approx \frac{G_F^2}{16\pi} f(s) \int_{-1}^1 [c_V^2(Q^2) + c_A^2(Q^2)] F(x) dx, \quad (9)$$

where

$$\begin{aligned} f(s) &= \frac{s(1 - M_{\tilde{H}}^2/s)}{[(1 - M_{\tilde{\chi}}^2/s)^2 - 2M_N^2 M_{\tilde{\chi}}^2/s^2]^{1/2}}, \quad F(x, s) = b + \frac{1}{4}(a_1 + bx)(a_2 + bx) - \\ &\quad \frac{M_{\tilde{\chi}} M_{\tilde{H}}}{s} b(1 - x); \quad a_{1,2} = 1 \pm \frac{M_{\tilde{\chi}}^2 - M_{\tilde{H}}^2}{s} - \frac{M_{\tilde{\chi}}^2 M_{\tilde{H}}^2}{s^2}; \quad b = (1 - \frac{M_{\tilde{\chi}}^2}{s})(1 - \frac{M_{\tilde{H}}^2}{s}); \\ Q^2 &= \frac{s}{2}(1 + \frac{M_{\tilde{\chi}}^2 - M_N^2}{s})(1 + \frac{M_{\tilde{H}}^2 - M_N^2}{s}) - \frac{s}{2} \bar{\lambda}(M_{\tilde{\chi}}^2, M_N^2; s) \bar{\lambda}(M_{\tilde{H}}^2, M_N^2; s) x \\ &\quad - M_{\tilde{\chi}}^2 - M_{\tilde{H}}^2; \quad \bar{\lambda}(a, b; c) = (1 - 2\frac{a+b}{c} + \frac{(a-b)^2}{c^2})^{1/2}; \quad x = \cos \theta. \end{aligned} \quad (10)$$

where  $\theta$  is the scattering angle. Note that the expressions (9) and (10) are not applicable if  $s \approx M_{\tilde{\chi}_1}^2$ , since our approximation ( $M_N = 0$  in the  $F(x, s)$ ) is violated in this regime. Note also that the cross-section in relativistic case depends on vector ( $c_V$ ) and axial-vector ( $c_A$ ) parts of the interaction. So, in this case the SI and SD contributions are mixed. From the expression for the value  $Q^2$  it follows that our approach is restricted by neutralino energy  $E_{\tilde{\chi}} \lesssim 2M_{\tilde{\chi}}$  for the “averaged” value of the scattering angle  $\theta$ . We have also derived the approximate formulae for the neutralino threshold energy (in laboratory frame of reference),  $E^{thr}$ , in the limit of small  $M_N \ll M_{\tilde{\chi}}$ :

$$E^{thr} = (M_{\tilde{\chi}_1} + M_{\tilde{H}}) \frac{\delta m^-}{2M_N} + M_{\tilde{H}}. \quad (11)$$

From this relation it follows that in the case  $M_{\tilde{\chi}_1} \sim 1$  TeV and  $\delta m^- \sim 1$  GeV the threshold energy is  $E^{thr} \sim 2M_{\tilde{H}}$ .

The typical value of the cross-section is about  $\sigma \sim 3 - 7$  fb for  $\delta m^- = 0.1$  GeV and  $E \sim 2 - 3$  TeV. It decreases with increasing of the mass splitting  $\delta m^-$  and the threshold energy  $E^{thr}$ . As it follows from the structure of  $\chi_1 \chi_2 Z$ -vertex, formula (9) is also valid in the case of the neutral channel process  $\chi_1 N \rightarrow \chi_2 N'$ . So, the cross-section of this scattering is also an order of few femtobarns at the considered energies. Thus, to detect the events of the neutralino-nucleon scattering we need a very massive and large-scale detector [43].

The cross-sections of the chargino-neutralino pair production at the LHC in all permitted combinations ( $\chi_1 \chi_2$ ,  $\chi_1 \tilde{H}$ ,  $\chi_2 \tilde{H}$ ,  $\tilde{H} \tilde{H}$ ) were given in Ref. [14] in the

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framework of analogous “low  $\mu$ ” Split SUSY model. For all processes the cross-sections are an order of  $10^{-3} - 10^{-5}$  pb with  $\mu = 800 - 1400$  GeV. Therefore, further we discuss the possible signatures of the final states only.

### 3. Decay properties of the light chargino

Now we turn to possible experimental signature of the neutralino-nucleon scattering. To this end we are going to analyze the decay channels of the products of scattering  $\tilde{H}$  and  $\chi_2$ . In particular, to study the recharge process  $\chi_1 n \rightarrow \tilde{H} p$  we have to deal with decays  $\tilde{H}^- \rightarrow \chi_1 l^- \bar{\nu}_l$ ,  $\chi_1 \pi^-$ ,  $\chi_1 \pi^- \pi^0$ , where  $l = e, \mu$  and  $\chi_1$  is LSP. The same decay modes define the final states signature in production processes of  $\tilde{H}$  at the LHC. Note that the three-pion decay mode of  $\tilde{H}$  is small for the mass-splitting  $\delta m^- \leq 1$  GeV.

The branching ratios of these channels strongly depend on the mass splitting  $\delta m^- = M_{\tilde{H}} - M_{\chi_1}$ . The  $\tilde{H}$  decay rate in semi-leptonic channels has the following form

$$\Gamma_l = \frac{G_F^2}{96\pi^3 M_{\tilde{H}}} \int_{M_l^2}^{(\delta m^-)^2} dq^2 \bar{\lambda}(q^2, M_{\chi_1}^2; M_{\tilde{H}}^2) \bar{\lambda}(0, M_l^2; q^2) [q^2 \bar{\lambda}^2(0, M_l^2; q^2) (M_{\tilde{H}}^2 + M_{\chi_1}^2 - 4M_{\tilde{H}} M_{\chi_1} - q^2) + (1 + \frac{M_l^2}{q^2} - 2\frac{M_l^4}{q^4}) ((M_{\tilde{H}}^2 - M_{\chi_1}^2)^2 - 2M_{\tilde{H}} M_{\chi_1} q^2 - q^4)], \quad (12)$$

where  $\delta m^- = M_{\tilde{H}} - M_{\chi_1}$  and  $\bar{\lambda}(a, b; c)$  is normalized Källén function defined in Eq. (10). Obviously, expression (12) differs from the analogous formula in Ref. [15, 16] for the case of light gaugino, but numerical results of calculation are close. Note also that analytical results coincides, if we rewrite a part of our expression in other kinematical variables and cast away some terms, which are subdominant numerically.

For  $l = e$  we have  $M_e \ll \delta m^-$ , and the expression for the corresponding decay width is simplified considerably

$$\Gamma_e \approx \frac{G_F^2}{192\pi^3} (\delta m^-)^5. \quad (13)$$

The decay rate of the process  $\tilde{H}^- \rightarrow \chi_1 \pi^-$  can be calculated with the help of the well known soft pion matrix element  $\langle \pi | \bar{d} \gamma_\mu (1 - \gamma_5) u | 0 \rangle = f_\pi q_\mu / \sqrt{2q^0}$ , where  $f_\pi \approx 132$  MeV is the pion decay constant and  $q$  is the four-momentum of  $\pi$ -meson. Substitution of this equality into the amplitude of the process  $\tilde{H} \rightarrow \chi_1 \bar{u} d$  leads to the decay rate in the one-pion channel

$$\Gamma_\pi \simeq \frac{G_F^2}{4\pi} |U_{ud}|^2 f_\pi^2 (\delta m^-)^2 M_{\tilde{H}} \sqrt{1 - 2\frac{M_\pi^2 + M_{\chi_1}^2}{M_{\tilde{H}}^2} + \frac{(M_\pi^2 - M_{\chi_1}^2)^2}{M_{\tilde{H}}^4}}. \quad (14)$$

In the channel with two final pions the transition from quark to hadron level is described by the matrix element [46, 47]

$$\langle \pi^- \pi^0 | \bar{d} \gamma_\mu (1 - \gamma_5) u | 0 \rangle = \sqrt{2} F_\pi(q^2) (k_- - k_+)_\mu, \quad (15)$$



where  $q = k_- + k_+$  is the sum of  $\pi^-$  and  $\pi^+$  momenta. Expression for  $F_\pi(q^2)$  can be taken from Refs. [46, 48]

$$\begin{aligned} F_\pi(q^2) &= \frac{M_\rho^2}{M_\rho^2 - q^2 - iM_\rho\Gamma_\rho(q^2)} \exp\left\{-\frac{q^2 \text{Re}[A(q^2)]}{96\pi^2 f_\pi^2}\right\}, \\ A(q^2) &= \log\left(\frac{M_\pi^2}{M_\rho^2}\right) + 8\frac{M_\pi^2}{q^2} - \frac{5}{3} + \sigma_\pi^3 \log\left(\frac{\sigma_\pi + 1}{\sigma_\pi - 1}\right), \quad \sigma_\pi = \sqrt{1 - 4\frac{M_\pi^2}{q^2}} \\ \Gamma_\rho(q^2) &= \theta(q^2 - 4M_\pi^2) \frac{\sigma_\pi^3 M_\rho q^2}{96\pi F_\pi^2}, \end{aligned} \quad (16)$$

where  $F_\pi = f_\pi/\sqrt{2} \simeq 93$  MeV (see Refs. [46, 48]). With the help of Eqs. (15) and (16) we get the two-pion decay rate as

$$\Gamma_{2\pi} = \frac{G_F^2 |U_{ud}|^2}{64\pi^3 M_{\tilde{H}}^2} \int_{q_1^2}^{q_2^2} |F_\pi(q^2)|^2 \sqrt{1 - 4\frac{M_\pi^2}{q^2}} f(q^2) \bar{\lambda}(M_{\chi_1}^2, q^2; M_{\tilde{H}}^2) dq^2, \quad (17)$$

where  $q_1 = 2M_\pi$ ,  $q_2 = \delta m^-$ ,  $F_\pi(q^2)$  is defined by Eq. (16), and

$$\begin{aligned} f(q^2) &= \frac{1}{6}(\delta m^-)^2 (M_{\tilde{H}} + M_{\chi_1})^2 - \frac{2}{3}M_\pi^2 (\delta m^-)^2 + \frac{8}{3}M_\pi^2 M_{\tilde{H}} M_{\chi_1} \\ &+ q^2 \left[ \frac{1}{6}(\delta m^-)^2 - \frac{2}{3}M_{\tilde{H}} M_{\chi_1} + \frac{4}{3}M_\pi^2 \right] - \frac{1}{3}q^4 - \frac{2M_\pi^2}{3q^2} (\delta m^-)^2 (M_{\tilde{H}} + M_{\chi_1})^2. \end{aligned} \quad (18)$$

Making use of strong inequalities  $M_\pi/M_{\tilde{H}} \ll 1$ ,  $\delta m^-/M_{\tilde{H}} \ll 1$ , the expression (17) can be simplified

$$\Gamma_{2\pi} \simeq \frac{G_F^2 |U_{ud}|^2}{48\pi^3} \int_{q_1^2}^{q_2^2} |F_\pi(q^2)|^2 \left(1 - \frac{4M_\pi^2}{q^2}\right)^{3/2} ((\delta m^-)^2 - q^2)^{3/2} dq^2 \quad (19)$$

Analogous formulae were represented in Ref. [15, 16] for the light gaugino decay. Changing the proper vertex functions, we get close numerical results in our case. However, analytical representations of the expressions for the decay rates are different.

Assuming  $\Gamma_H^{tot} \approx \sum_l \Gamma_l + \Gamma_\pi + \Gamma_{2\pi}$ , we can evaluate the branching ratios  $B_l = \Gamma_l/\Gamma_H^{tot}$ ,  $B_\pi = \Gamma_\pi/\Gamma_H^{tot}$  and  $B_{2\pi} = \Gamma_{2\pi}/\Gamma_H^{tot}$ , which describe the signature of the total chargino decay process. Calculated ratios are presented in Fig. 2 as functions of the mass splitting  $\delta m^-$  at fixed  $M_{\tilde{H}} = 1.4$  TeV.

We see from Fig. 2 that the branching ratio of the pion channel is strongly dominant above the threshold due to soft nature of the pion production. With the increase of the mass splitting  $\delta m^-$ , the processes go to the hard limit, and  $\text{Br}(\pi)$  becomes an order of  $\text{Br}(l\bar{\nu}_l)$ , that is, an order of QCD limit  $\text{Br}(d\bar{u})$ . The two-pion decay mode is the most significant one at  $\delta m^- \approx 1$  GeV. Furthermore, we neglect the channels with production of  $K, \rho, \dots$  mesons in the final state. This is because the channel with  $K^-$ -meson in the final state is suppressed by the factor  $|U_{ds}|^2 \approx 0.05$ , while decay into  $\rho$  meson leads to the two-pion final state. So, we simulate all hadronic decay modes of chargino by pion final states. Certainly, in the

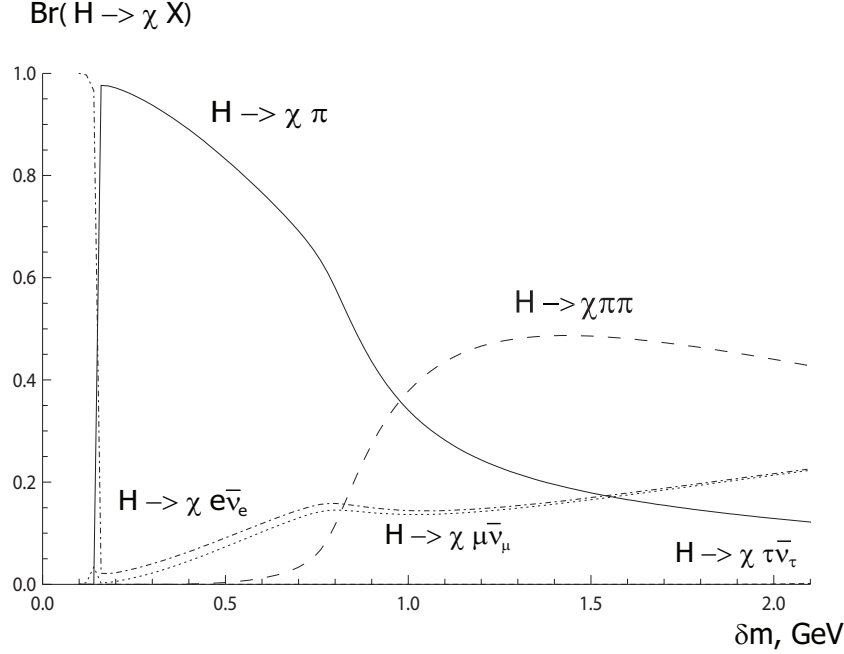


Fig. 2. Branching ratios for chargino decays as functions of mass splitting.

QCD region these reactions should be described by the amplitude  $\tilde{H} \rightarrow q' \bar{q} \chi_1$ , as it was also considered quantitatively in Ref. [18] in the SUSY scenario with  $\mu \gg M_{1,2}$ .

#### 4. Decay properties of the next lightest neutralino

Now let us consider the neutral channel of the scattering  $\chi_1 N \rightarrow \chi_2 N'$  with the consequent decay of  $\chi_2$ . Because of ambiguity concerning the neutralino mass splitting, we consider  $\delta m \sim \delta m^-$ . As it was mentioned before, formula for the cross-section is the same as for the recharge process. However, signature of the total decay process is different. In this case, we have the dominant decay channels  $\chi_2 \rightarrow \chi_1 f \bar{f}$ , where  $f = e, \mu, \nu$  and  $\chi_2 \rightarrow \chi_1 \pi^0$   $\chi_2 \rightarrow \chi_1 \pi^+ \pi^-$ . The semi-leptonic decay rate is

$$\begin{aligned} \Gamma_l = & \frac{G_F^2}{64\pi^3 M_{\chi_2}} \int_{4M_l^2}^{(\delta m)^2} dq^2 \bar{\lambda}(q^2, M_{\chi_1}^2; M_{\chi_2}^2) \bar{\lambda}(M_l^2, M_l^2; q^2) \left\{ \frac{c_+}{6} [q^2 \bar{\lambda}^2(M_l^2, M_l^2; q^2) \times \right. \\ & \times (M_{\chi_2}^2 + M_{\chi_1}^2 - 4M_{\chi_2} M_{\chi_1} - q^2) + (1 + 2\frac{M_l^2}{q^2}) ((M_{\chi_2}^2 - M_{\chi_1}^2)^2 \\ & \left. - 2M_{\chi_2} M_{\chi_1} q^2 - q^4)] + c_- M_l^2 (M_{\chi_2}^2 + M_{\chi_1}^2 - 4M_{\chi_2} M_{\chi_1} - q^2) \right\} \end{aligned} \quad (20)$$

where  $c_{\pm} = c_V^2 \pm c_A^2$ , and  $\delta m = M_{\chi_2} - M_{\chi_1}$ . In particular, for the neutrino channel ( $f = \nu$ ) we have

$$\Gamma_{\nu} = \frac{G_F^2}{3 \cdot 256\pi^3} (\delta m)^5. \quad (21)$$

The decay rate of the pion neutral channel is found in complete analogy with the charge channel and reads

$$\Gamma_\pi \simeq \frac{G_F^2}{8\pi} f_\pi^2 (\delta m)^2 M_{\chi_2} \sqrt{1 - 2 \frac{M_\pi^2 + M_{\chi_1}^2}{M_{\chi_2}^2} + \frac{(M_\pi^2 - M_{\chi_1}^2)^2}{M_{\chi_2}^4}}. \quad (22)$$

And the corresponding decay rate of the two-pion neutral channel in analogy with Eq. (19) is

$$\Gamma_{2\pi} \simeq \frac{G_F^2}{96\pi^3} \int_{q_1^2}^{q_2^2} |F_\pi(q^2)|^2 \left(1 - \frac{4M_\pi^2}{q^2}\right)^{3/2} ((\delta m)^2 - q^2)^{3/2} dq^2 \quad (23)$$

where  $q_1 = 2M_\pi$ ,  $q_2 = \delta m$  and  $F_\pi(q^2)$  is defined by Eq. (16). Similarly to the charged case, for the neutral scattering the signature is represented by the branching ratios of the neutralino decay channels,  $B_l = \Gamma_l/\Gamma^{tot}$ ,  $B_\pi = \Gamma_\pi/\Gamma^{tot}$  and  $B_{2\pi} = \Gamma_{2\pi}/\Gamma^{tot}$ , which are represented in Fig. 3. These ratios are functions of the mass-splitting  $\delta m$  at fixed  $M_{\chi_1} = 1.4$  TeV.

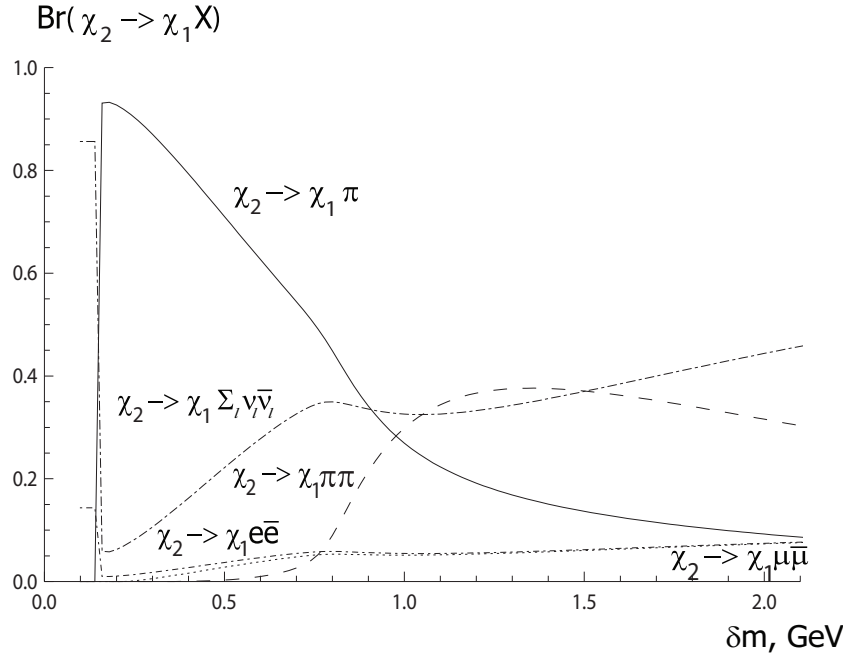


Fig. 3. Branching ratios for neutralino decays as functions of mass splitting.

From Fig. 3 we conclude that behavior of the branching ratios in the neutral case is similar to that for the chargino decays. Namely, here again the process of pion production starts in the soft regime and goes to the QCD limit (hard regime) with increasing of the mass splitting  $\delta m$ . We should also mention that the decay channel with  $K^0$ -meson is strongly suppressed due to absence of  $Zds$  vertex at the tree level.

However, in analogy with the chargino decay, the channels with  $2\pi$  production is quite noticeable at  $\delta m \approx 1$  GeV.

Collecting all results for the chargino and neutralino decay channels in the scenario under consideration, we get the chargino  $\tilde{H}$  and neutralino  $\chi_2$  ranges,  $l = c\tau$  ( $\tau$  is the life time of the  $\tilde{H}$  or  $\chi_2$ ), depending on  $\delta m^-$  and  $\delta m$ , respectively. The corresponding curves are shown in Fig. 4 (see, for comparison, Refs. [15, 16, 18]). One can see that the chargino track can be detected if the small mass splitting  $\delta m^-$  occurs.

From the above presented results it follows that if  $\delta m^- \lesssim m_\pi$ , we have no any visible signals from the chargino decay  $\tilde{H} \rightarrow l\nu\chi_1$  due to the final leptons softness. Further, with the increasing of the mass splittings, when  $m_\pi < \delta m^-$ ,  $\delta m \lesssim 1$  GeV, chargino and the NLSP decay mainly through one-pion and two-pion channels (see Figs. 2 and 3). The charged pions can be visible in experiment together with the chargino track. For the NLSP with  $\delta m \sim 1$  GeV the neutrino channels are very important, i.e. fraction of the events with large missing energy increases up to almost 50 %. In Refs. [13, 14, 15, 16, 18], the analogous analysis of possible final states was made for the case with the degenerated lowest chargino and neutralino states. However, this analysis was fulfilled for the SUSY scenario with the relatively light neutralino and chargino states,  $M_\chi \sim O(100 \text{ GeV})$ . Moreover, our results for the chargino and neutralino branching ratios are different from the ones in papers mentioned above (cf. Figs. 2 and 3 and corresponding curves from these papers).

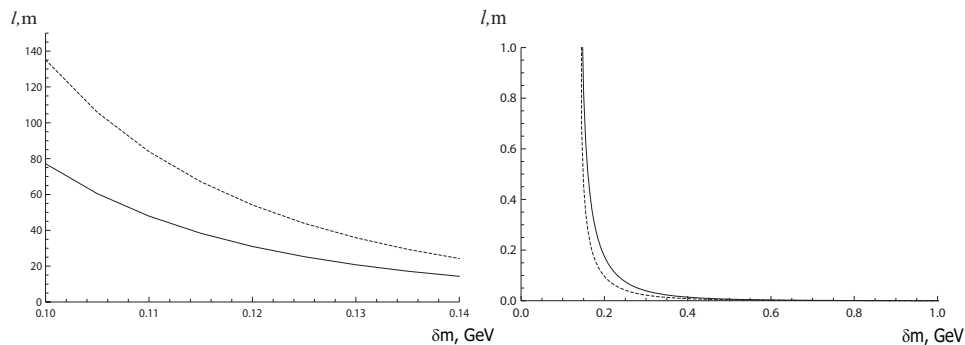


Fig. 4. The range of neutralino (solid line) and chargino (dashed line) as function of mass splitting.

It should be mentioned that charged Higgs bosons are arranged at some high-energy scale, as it specifically takes place in Split SUSY models (despite of one “standard” neutral Higgs boson). Then, all additional contributions to the considered processes, which are mediated by the heavy Higgs bosons, are suppressed. The same suppression occurs in reactions with intermediate heavy squark (slepton) states.

## 5. Conclusions

The one-loop RG analysis of the SUSY  $SU(5)$  theory leads to a few sets of energy scales, which are compatible with the conventional ideas on the DM structure and experimental expectations. Due to a specific form of the RG equations the super-scalar scale  $M_0$  remains arbitrary, and it occurs that the variety of possible scenarios can be divided into two classes:  $M_{1/2} \gg \mu$  or  $M_{1/2} \ll \mu$ .

In this paper, the hierarchy  $M_0 \gtrsim M_{1/2} \gg \mu$  (the Split Higgsino model) and its possible experimental manifestations were considered. For this case, the (one-loop) RG approach resulted to the SUSY breaking scale  $M_{SUSY} \sim 10^8 - 10^9$  GeV and neutralino mass in the interval  $1.2 - 1.6$  TeV.

We have considered the neutralino-nucleon scattering at low (relic neutralino) and high energies. The high energy neutralino can be produced as a result of high energy cosmic ray annihilation or in decays of an exotic super-heavy  $M_X$  particles. It was shown that the energy of relic neutralino is far below the threshold in the scenario under consideration. The scattering of high energy neutralino off nucleon target can produce the nearest SUSY states,  $\chi_2$  and/or chargino  $\tilde{H}$ . We have analyzed in details the signature of their decays, which make it possible to extract these events from the background caused mainly by the neutrino-nucleon scattering.

From the analysis we have fulfilled, it follows that in order to detect some foot-steps of Split Higgsino scenario, one have to analyze the correlation of collider data,  $\chi - N$  cross section measurements and value of diffuse gamma flux from halo (or direct photon spectrum). Only the comparison of all measured characteristics could provide us with some conclusions on the particular realization of the Split SUSY scenario. In a sense, this model presents a class of “Hidden SUSY” scenarios which do not reject SUSY ideas and, at the same time, can explain (possible) absence of obvious SUSY signals at the LHC in the TeV region.

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## Appendix A.

Here we briefly analyze connection between the sign of neutralino masses and structure of neutralino-bozon interaction. The limit  $M_Z/M_k \rightarrow 0$ , where  $M_k$  is  $M_1, M_2$  or  $\mu$ , allows to simplify the analysis which can be used in the general case too.

If we omit the mixing of gauge and Higgs fermions, the mass term of higgsino-like Majorana fields has the Dirac form:

$$M_h = \frac{1}{2} \mu (\bar{H}_{1R}^0 H_{2L}^0 + \bar{H}_{2R}^0 H_{1L}^0) + h.c. . \quad (A.1)$$

This form can be represented by a  $(2 \times 2)$ - mass matrix having zero trace:

$$\mathbf{M}_2 = \begin{pmatrix} 0 & \mu \\ \mu & 0 \end{pmatrix}. \quad (A.2)$$

There are two ways to diagonalize this matrix. The formal procedure using the orthogonal matrix  $\mathbf{O}_2$  leads to a spectrum with opposite signs:

$$\mathbf{O}_2^T \mathbf{M}_2 \mathbf{O}_2 = \begin{pmatrix} \mu & 0 \\ 0 & -\mu \end{pmatrix}, \quad \mathbf{O}_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \quad m_a = (\mu, -\mu). \quad (\text{A.3})$$

In this case, one of the Majorana fields has a negative mass that is followed from the trace conservation  $\text{Tr}\{\mathbf{O}_2^T \mathbf{M}_2 \mathbf{O}_2\} = \text{Tr}\{\mathbf{M}_2\} = 0$ .

The matrix  $\mathbf{M}_2$  can also be diagonalized by the unitary complex matrix  $\mathbf{U}_2$ , giving masses with the same sign

$$\mathbf{U}_2^T \mathbf{M}_2 \mathbf{U}_2 = \begin{pmatrix} \mu & 0 \\ 0 & \mu \end{pmatrix}, \quad \mathbf{U}_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ 1 & -i \end{pmatrix}, \quad m_a = (\mu, \mu). \quad (\text{A.4})$$

Obviously, relation  $H_L^0 = (H_R^0)^C$  for the Majorana spinor  $H^0$  leads to the first term in (A.4). This relation defines procedure of diagonalization of Majorana mass forms in the general case. The diagonalization (A.4) is equivalent to the procedure (A.3) with the redefinition  $\chi \rightarrow i\gamma_5 \chi$  of the non-chiral (full) field with  $m = -\mu$  (for the chiral components it corresponds to the transformation  $\chi_{R,L} \rightarrow \pm i\chi_{R,L}$ ).

In this case, however, there is an infinite set of unitary matrices  $\mathbf{U}_\phi = \mathbf{U}_2 \cdot \mathbf{O}_\phi$  which diagonalize the mass matrix  $\mathbf{M}_2$  (see [31] and reference therein):

$$\mathbf{U}_\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{i\phi} & ie^{i\phi} \\ e^{-i\phi} & -ie^{-i\phi} \end{pmatrix}, \quad \mathbf{O}_\phi = \begin{pmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{pmatrix}. \quad (\text{A.5})$$

It has been shown in Ref. [31] that the additional  $O_2$ -symmetry leads to a free parameter arising in the general case.

Dealing with the spinor field we should take into account the sign of its mass in the propagator and polarization matrix or redefine the field with a negative mass. As a rule this feature is not considered in numerous phenomenological applications (see, [31] and references therein). The redefinition of the Majorana spinor  $\chi' = i\gamma_5 \chi$  changes the sign of mass in the term  $m\bar{\chi}\chi$  and does not change the term  $i\bar{\chi}\gamma^k \partial_k \chi$ . This spinor transformation saves the Majorana condition  $(i\gamma_5 \chi)^C = i\gamma_5 \chi$  too. Note also that for non-chiral Majorana field the redefinition  $\chi' = i\chi$  is not permissible. From the redefinition  $\chi' = i\gamma_5 \chi$  it follows that the transformation properties (relative to inversion) of Majorana fields having opposite mass signs are different. It results to one usual Majorana field and one pseudo-Majorana field. So, the mass signs are directly linked with the relative parity, and this is important for the correct interpretation of  $Z\chi_1\chi_2$  interaction.

The gaugino mass subform is of the standard Majorana type and the signs of the masses for  $\chi_3$  and  $\chi_4$  are defined by the signs of  $M_1$  and  $M_2$  in the case of small mixing. They can be made positive by the appropriate redefinition.

Let us consider for simplicity the pure higgsino approximation to analyze the connection between the structure of boson-neutralino interaction and the relative sign of neutralino masses. It can be seen that the calculation rules should be different in two cases – when masses of  $\chi_2$ ,  $\chi_1$  have opposite signs (diagonalization (A.3)) and when they have the same signs (diagonalization (A.4)).

The initial Lagrangian is

$$L_{int} = \frac{1}{2}g_Z Z_\mu (\bar{H}_{1L}^0 \gamma^\mu H_{1L}^0 + \bar{H}_{2R}^0 \gamma^\mu H_{2R}^0). \quad (\text{A.6})$$

where  $g_Z = g_2/\cos\theta_W$ . The diagonalizations (A.3) and (A.4) lead to the following forms of neutralino-boson interactions, respectively:

$$(1) L_{int} = -\frac{1}{2}g_Z Z_\mu \bar{\chi}_2 \gamma^\mu \gamma_5 \chi_1'; \quad (2) L_{int} = \frac{i}{2}g_Z Z_\mu \bar{\chi}_2 \gamma^\mu \chi_1. \quad (\text{A.7})$$

In Eqs. (A.7) the first case, having opposite signs  $(\mu, -\mu)$ , can be transformed into the second case with the same signs  $(\mu, \mu)$  by the redefinition  $i\gamma_5 \chi_1' = \chi_1$ . It was shown in Ref. [31] that both forms of  $L_{int}$  in Eqs. (A.7) give the same result without any field redefinition if the negative sign of  $\chi_1'$  mass is taken into account in calculations evidently. So, both structures in Eqs. (A.7) lead to the parity-conserving vector interaction giving the spin-independent contribution to the neutralino-nucleon scattering. It is provided by the pseudo-Majorana nature of the  $\chi_1'$  field.

As it has been shown in Ref. [31], we cannot draw any reasonable conclusions on the SD or SI contributions from the interaction Lagrangian only, without consideration of the mass signs. In other words, calculation rules should correlate with the signs of neutralino masses. Specifically, the bilinear structures  $\bar{\chi}_2 \gamma_\mu \chi_1$  and  $\bar{\chi}_2 \gamma^\mu \gamma_5 \chi_1'$  are vectors, while  $\bar{\chi}_2 \gamma_\mu \chi_1'$  and  $\bar{\chi}_2 \gamma^\mu \gamma_5 \chi_1$  are axial vectors. The structures  $\bar{\chi}_2 \chi_1'$  and  $\bar{\chi}_2 \chi_1$  are pseudoscalar and scalar, respectively. We conclude that the analysis of the neutralino-nucleon interaction has to take into account neutralino transformation properties. In particular, for the current structure  $\bar{\chi}_i \gamma^\mu \gamma_5 \chi_k Z_\mu$  it is possible to obtain SD or SI neutralino-nucleon cross sections depending on the neutralino relative parity.

## Appendix B.

In this Appendix we represent some technical details of calculations, which are important to get the results above.

Let us consider the  $\chi_1 N$  scattering in the non-relativistic limit (see Fig. 1). The needed vertexes are described by the Lagrangians:

$$L_{Z\chi} = \frac{ig}{2\cos\theta_W} Z_\mu \bar{\chi}_2 \gamma^\mu \chi_1, \quad L_{ZN} = \frac{ig}{4\cos\theta_W} Z_\nu \bar{N} \gamma^\nu (c_V - c_A \gamma_5) N. \quad (\text{B.1})$$

At  $q^2 \approx 0$  we use  $c_V = 1$ ,  $c_A = 1.25$  for the neutron and  $c_V = 1 - 4\sin^2\theta_W$ ,  $c_A = 1.25$  for the proton. The process  $\chi_1 N \rightarrow \chi_2 N$  in the  $t$ -channel has an amplitude in accordance with Eq. (B.1):

$$M = \frac{ig^2}{8\cos^2\theta_W M_Z^2} \bar{\chi}_2(k_2) \gamma^\mu \chi_1(p_1) \cdot \bar{N}(p_2) \gamma_\mu (c_V - c_A \gamma_5) N(k_2), \quad (\text{B.2})$$

where  $p_i$  and  $k_i$  are the four-momenta of particles. Note, as it was shown in Refs. [32, 39], in the non-relativistic case the vector part of  $ZN$  vertex ( $c_V$ ) gives spin-

independent (SI) contribution, while the axial-vector part of vertex ( $c_A$ ) gives spin-dependent contribution. Further we show that the vector type of the  $Z\chi_1\chi_2$  interaction cut off in the non-relativistic case the vector part of  $ZN$  vertex and the cross-section in this case depends on  $c_V$  only. By the standard straightforward calculation we get in the non-relativistic limit:

$$|M|^2 \approx K\{(c_V^2 + c_A^2)[(p_1 p_2)(k_1 k_2) + (p_1 k_2)(p_2 k_1)] - 2c_A^2 M_N^2 (p_1 k_1)\}, \quad (\text{B.3})$$

where  $K$  is some numerical coefficient. Because the cross-section value is proportional to the small momenta of  $\chi_2$  and  $N$  in the CMS, we can use an approximation  $(p_1 p_2) \approx M_{\chi_1} M_N$ ,  $(k_1 k_2) \approx M_{\chi_2} M_N$  etc. in the expression (B.3). As a result, we have  $|M|^2 \approx 2K c_V^2 M_{\chi_1} M_{\chi_2} M_N^2$ , so the SI term only survives in the case under consideration. Then, standard calculations give a simple formula for the cross-section:

$$\sigma \approx \frac{G_F^2}{8\pi v_r} c_V^2 M_N k, \quad (\text{B.4})$$

where  $k$  is absolute value of three-momenta of  $\chi_2$  and  $N$  in the SCI, which is usually defined by the Källen function

$$k = \frac{\sqrt{s}}{2} \left(1 - 2 \frac{M_N^2 + M_1^2}{s} + \frac{(M_N^2 - M_2^2)^2}{s^2}\right)^{1/2}. \quad (\text{B.5})$$

Here  $s = (p_1 + p_2)^2 = (k_1 + k_2)^2$  and  $M_{1,2} = M_{\chi_{1,2}}$ . The value of  $k$  can be found from the expressions for  $s$  in the laboratory coordinate system (LCS) and center-of-mass system (CMS):

$$\begin{aligned} s &= (p_1 + p_2)^2 \approx (M_1 + M_N)^2 + M_1 M_N v_r^2, \quad (\text{LCS}); \\ s &= (k_1 + K_2)^2 \approx (M_2 + M_N)^2 + k^2 \frac{M_2}{M_N}, \quad (\text{CMS}). \end{aligned} \quad (\text{B.6})$$

From these equations it follows:

$$k = \sqrt{\frac{M_N}{M_2}} \{M_1 M_N v_r^2 - 2\delta m (M_1 + M_N)\} \approx \sqrt{2M_N} (W_1 \frac{M_N}{M_1} - \delta m)^{1/2}, \quad (\text{B.7})$$

where  $\delta m = M_2 - M_1$  and  $W_1 = M_1 v_r^2/2$  is the kinetic energy of neutralino  $\chi_1$  in LCS ( $v_r$  is the dimensionless relative velocity in units of the speed of light  $c$ ). Finally, we get formula for the non-relativistic neutralino-nucleon scattering:

$$\sigma = \frac{G_F^2}{4\sqrt{2}\pi} \frac{c_V^2}{v_r} M_N^{3/2} (W_\chi \frac{M_N}{M_\chi} - \delta m)^{1/2}. \quad (\text{B.8})$$

Thus, the neutralino-nucleon scattering has two features — the main contribution is spin-independent and the threshold of the reaction is  $W_\chi^{thr} = \delta m \cdot (M_\chi/M_N)$ .

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